SUBSURFACE WATER AND FLOW PROCESS

By

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Introduction

Leonard Davinci had stated, “The greatest river of the earth flows underground”. Ground water can be tapped almost everywhere digging a well and water can be lifted to ground surface spending external energy. Unlike surface water, most groundwater moves very slowly. The slow movement of groundwater is advantageous in respect of its availability in a region. In subsurface saturated zone, it flows from the region of recharge to a favorable discharge region. A spring is a typical example of groundwater outlet. The lean flow in a stream is supported by groundwater flow.

Groundwater System

The groundwater system may be imagined as a huge natural reservoir or system of reservoirs in rocks whose capacity is the total volume of pores or openings that are filled with water. Groundwater may be found in one continuous body or in several distinct rock or sediment layers at any one location. Thickness of the groundwater zone is governed by the local geology, availability of pores or openings in the rock formation, recharge, and movement of water from areas of recharge towards points or areas of discharge.

It is nearly impossible to adequately summaries all types of geologic environments in which water can exit, but the list below presents some typical types of openings found in rocks (Driscoll, 1987):

(i) Inter-grain pores in unconsolidated sand and gravel, pores in sandstone, and shale;

(ii) Systematic joints in metamorphic and igneous rocks;

(iii) Cooling fractures, gas-bubble holes and lava tubes in basalt;

(iv) Solution cavities and systematic joints in limestone;

(v) Openings in fault zones.

In nature, rock masses are rarely homogeneous and adjacent rock types may vary significantly in their ability to hold water.

Aquifers

An aquifer is a saturated bed, formation, or group of formations, which yields water in sufficient quantity to be economically useful. Water-bearing formations and groundwater reservoirs are synonyms for the word aquifer. To be an aquifer, a geologic formation must contain pores or open spaces (both of these are often called interstices) that are filled with water. These interstices must be large enough to transmit water towards wells at a useful rate.

A non-dimensional parameter, K_c, designated as conductivity class has been defined (Bear and Verruijt, 1987) by the relation

$$K_c = -[2 + \log_{10} k]$$

in which k is the hydraulic conductivity in m/s. K_c is used as an index to classify the aquifers and the porous media as given in Table-1.

Table -1. Classification of porous media/aquifer using conductivity class (K_c)

Porous Media/Aquifer	
Pervious	-2 to 2
Semi pervious	2 to 6
Impervious	6 to 11
Good aquifer	-2 to 3
Poor aquifer	3 to 7

Although clay has a large water holding capacity, water cannot move readily through the tiny open spaces. This means that a clay formation under normal conditions will not yield water to wells, and therefore it is not an aquifer even though it may be water-saturated.

Ordinarily a clay or shale formation is nearly impermeable and is called an aquiclude, or a formation through which virtually no water moves. Formations, which do yield some water, but usually not enough to meet even modest demands, are called aquitards. In reality, almost all formations will yield some water, and therefore are classified as either aquifers or aquitards. In water poor areas, a formation producing small quantities of water may be called an aquifer, whereas the same formation in a water-rich area would be an aquitard.

Aquifers have two main functions in the underground phase of the water cycle. They store water for varying periods in the underground reservoir, and they act as pathways or conduits to pass water along through the reservoir. Although some are more efficient as pipelines (e.g., cavernous limestones) and some are more effective as storage reservoirs (e.g., sandstones), most aquifers perform both functions continuously.

Type of Aquifers

Confined and Unconfined Aquifers

Aquifers may be classified as unconfined or confined depending on the presence or absence of a water table. For an unconfined aquifer a water table serves as the upper surface of the zone of saturation. The water table is defined as “ that surface in the groundwater body at which the water pressure is atmospheric". Unconfined groundwater, then, is water in an aquifer that has a water table in contact with the atmosphere through pores in the unsaturated soil above. Unconfined aquifers are sometimes called water table aquifers.

Confined groundwater, on the other hand, is water under pressure greater than atmospheric pressure. The upper boundary of a confined aquifer is essentially impermeable formation that “traps” or “confines” water in the aquifer, sealing it off from the atmosphere.

When a well is drilled in an unconfined aquifer, water will remain approximately at the same level where it is first encountered. In wells tapping a confined aquifer, the water will rise in the well when it is first encountered during drilling, and will stand at a level above the top of the aquifer.

Water in a confined aquifer is under pressure because the aquifer is at a higher elevation in the recharge area than it is at the well location. Depending on local conditions, water from a confined aquifer will even rise up in the well until it flows at the surface without the aid of a pump. Traditionally such a well is called an artesian well. Confined aquifers are often called artesian aquifer.

Storage in Unconfined Aquifers

An unconfined aquifer’s pore space contains both live storage and dead storage. Gravity water makes up the live storage and capillary water the dead storage. When the water table rises, it saturates the overlying capillary fringe; when it falls, it leaves capillary water suspended above it. In either case only gravity water actually moves into or leaves the storage in the aquifer, causing water levels in wells to rise or fall. For an unconfined aquifer, the ratio of volume of capillary water to its own volume (aquifer volume) that contains it is called specific retention; the ratio of aquifer volume containing gravity water that is free to move in or out of the pores to its total volume is called specific yield. Specific retention plus specific yield equals porosity, which is the ratio of total open space to total volume of the aquifer.

Storage in Confined Aquifers

The implications for changes in storage as piezometric levels fluctuate are very different for confined versus unconfined aquifers. In an unconfined aquifer, lowering the water table actually dewaters the upper part of the aquifer and reduces the volume of saturated material, but the aquifer and the water undergo essentially no physical changes. When water levels fall in wells tapping a confined aquifer, the pressure falls, but the aquifer remain completely saturated. The change in pressure in a confined aquifer affects both the water and the aquifer.

The aquifer responds to pressure change by expanding or contracting slightly as water levels rise or lower. This produces a small increase or decrease in porosity, and hence in storage space. Water, although usually thought of as being incompressible, will also contract or expand slightly in response to pressure changes. Thus, when water is removed from storage, well levels fall, pressure in the aquifer falls, water expands slightly, and the aquifer contracts slightly. When water is added to storage, the reverse process takes place: well levels rise, pressure in the aquifer rises, water contracts slightly and the aquifer expands slightly. Because the change in aquifer porosity or in water volume per unit change in pressure is very small, the numerical value of the storage coefficient for an artesian aquifer is much smaller than that for a water-table aquifer. The range in values for the storage coefficient in water–table aquifers is around 0.01-0.03 while the storage coefficients for artesian aquifers are more like 0.00005-0.005 (Manning , 1989).

Bernoulli's equation

For steady flow of non-viscous incompressible fluids Bernoulli's equation is:

$$\frac{p}{\gamma_w} + z + \frac{\bar{v}^2}{2g} = constant = h$$

where, p=fluid pressure (N/m^2 ; γ_w =unit weight of water (N/m^3); $\bar{v}$ = seepage velocity i.e. true velocity of fluid through pores in case of groundwater flow(m/s); g=acceleration due to gravity(m/s^2), h= total head (m)=energy per unit weight of fluid(m). To take into account loss of energy due to viscous resistance within pores, Bernoulli's equation is expressed as:

$$\frac{p_1}{\gamma_w} + z_1 + \frac{\bar{v}_1^2}{2g} = \frac{p_2}{\gamma_w} + z_2 + \frac{\bar{v}_2^2}{2g} + \Delta h$$

points 1 and 2 are two points on a stream tube at a distance Δs apart; Δh is the head loss between the two points 1 and 2 (Figure1). The above equation implies that the flow is in a steady state condition.

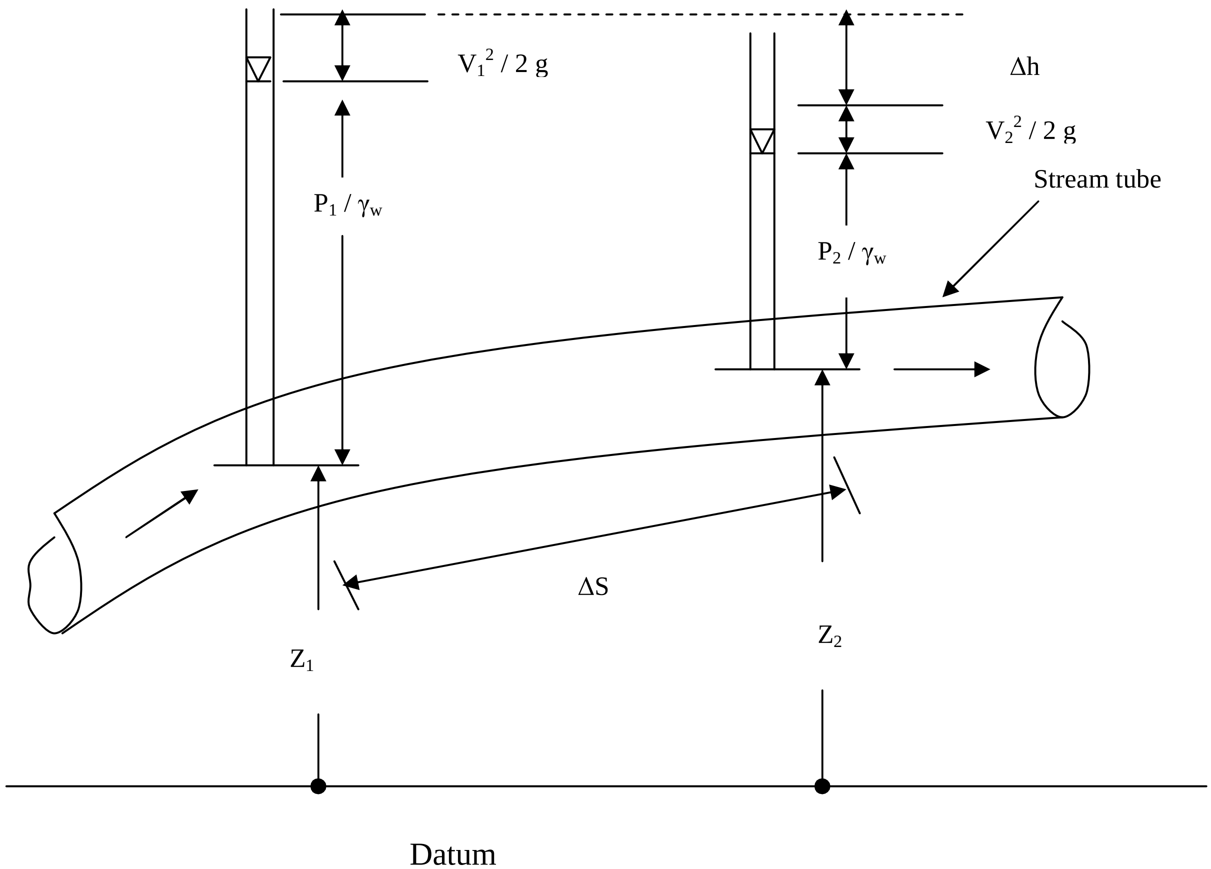

Fig. 1 Steady flow of water in a stream tube

Darcy's Law

Based on experiments Darcy stated the following law:

The discharge per unit of pore plus soil area, which is known as discharge velocity, in a homogeneous isotropic porous medium is given by:

$$v = -k\frac{\partial h}{\partial s}$$

$\frac{\partial h}{\partial s}$ = variation in hydraulic head along flow direction 's'; k=coefficient of permeability. The Darcy's law demonstrates that the discharge velocity is linearly proportional to the hydraulic gradient. The true velocity $\bar{v}$ known as seepage velocity is $\frac{v}{n}$ where 'n' is volumetric porosity of the medium equal to average area porosity. A tracer injected to the flow domain moves with true velocity. The hydraulic head in ground water flow excludes the velocity head $\frac{\bar{v}^2}{2g}$. The hydraulic head *h* is defined as $h = \frac{p}{\gamma_w} + y$ where *p* is pore water pressure, γ_w is unit weight of water, *y* is elevation head measured positive upward.

Darcy's Law in Homogeneous Anisotropic Medium

If the properties of the soil that are responsible for the resistance to flow are independent of the direction, such a material is said to be isotropic with regard to permeability. Not every soil possesses that property, however. In many soil deposits the resistance to flow in the vertical direction is considerably larger than the resistance to horizontal flow, due to the presence of layered structure in the soil, generated by its geological history. For such anisotropic porous media Darcy's law has to be generalized. The proper generalization is, in term of hydraulic conductivity (Verruijt, 1982).

$$q_x = -k_{xx}\frac{\partial h}{\partial x} - k_{xy}\frac{\partial h}{\partial y} - k_{xz}\frac{\partial h}{\partial z}$$

$$q_y = -k_{yx}\frac{\partial h}{\partial x} - k_{yy}\frac{\partial h}{\partial y} - k_{yz}\frac{\partial h}{\partial z}$$

$$q_z = -k_{zx}\frac{\partial h}{\partial x} - k_{zy}\frac{\partial h}{\partial y} - k_{zz}\frac{\partial h}{\partial z}$$

The quantities q_x, q_y and q_z are the three components of the specific discharge vector q, where specific discharge vector denotes the discharge through a certain area of soil divided by that area. $\partial h/\partial x$, $\partial h/\partial y$ and $\partial h/\partial z$ are components of the hydraulic head gradient in x, y, z direction respectively.

These equations express the most general linear relationship between the specific discharge vector and the gradient of the groundwater head. It is usually assumed that $k_{xy} = k_{yx}$, $k_{yz} = k_{zy}$, $k_{zx} = k_{xz}$). It can be shown that this means that there exist three mutually orthogonal directions, the so-called principal directions of permeability, in which the cross-components vanish. Physically speaking this means that a gradient of the groundwater head in one of these directions leads to a flow in that same direction.

In general the anisotropy law should be of the form of equation (1), with six independent coefficients. Fortunately in engineering practice it is usually acceptable to distinguish only between the permeability in vertical direction and one in horizontal direction, assuming that this difference has been created during the geological process of deposition of the soil. Then it may assumed that the x,y,z-directions are principal directions (if the z-axis is vertical), with $k_{xx} = k_{yy} = k_h$ and $k_{zz} = k_v$. Darcy’s law can then be used in the form

$$q_x = -k_h \frac{\partial h}{\partial x}$$

$$q_y = -k_h \frac{\partial h}{\partial y}$$

$$q_z = -k_v \frac{\partial h}{\partial z}$$

which involves only two coefficients. They must be measured by doing two independent tests.

Range of Validity of Darcy’s Law

Several investigators have found that flow in soils changes from laminar to turbulent for $1 \le R_e < 12$ (vide Harr,1962) where the Reynolds’ number in case of flow through soils is given by $R_e = \frac{vd\rho}{\mu}$, v=discharge velocity, d=average of diameters of soil particles, $\rho =$ density of fluid, and μ

=coefficient of viscosity. Darcy's law is accepted to be valid when Reynolds number R is equal to or less than 1.

For a fully turbulent condition, the relation between hydraulic gradient and velocity of flow is represented as:

$$-i = av + bv^2 \tag{1}$$

where 'a' , and 'b' are positive constants and $i = \frac{dh}{ds}$. Equation (1) is valid for which $\frac{dh}{ds}$ is negative and v is positive. Solving 'v' from above,

$$v = \frac{-a}{2b} + \frac{\sqrt{a^2 - 4bi}}{2b} = \frac{-a}{2b} + \frac{a}{2b}\left[1 - \frac{4bi}{a^2}\right]^{1/2} \cong -\frac{i}{a} - \frac{bi^2}{a^3} \tag{2}$$

Under turbulent flow condition the velocity for a given gradient is less than the velocity that would result under laminar flow condition. Under turbulent flow condition the specific capacity of a well is less than that under laminar flow condition. The following example explains the effect of turbulence on specific capacity.

Example

A fully penetrating well with radius r_w in a confined aquifer at the center of a circular groundwater basin having a constant head boundary condition at the outer periphery is recharged maintaining a constant head at the well face. Find the recharge rate per unit water level rise at the well face?

Under steady state flow condition, at any radial distance ‘r’ from the well, the radial flow is given by:

$$Q_r = 2\pi\, rDv_r = Q_R \tag{3}$$

where D=thickness of aquifer, v_r = radial Darcy’s velocity. From (1),

$$v_r = \frac{-a}{2b} + \frac{\sqrt{a^2 - 4b\dfrac{dh}{dr}}}{2b} \tag{4}$$

Incorporating v_r in (3) after simplification

$$\left(\frac{bQ}{\pi D}\right)^2 \frac{1}{r^2} - \left(\frac{2abQ}{\pi D}\right)\frac{1}{r} = -4b\frac{dh}{dr}$$

Integrating and applying the boundary conditions $h(r_w) = h_w$, and $h(R) = h_R$

$$h_w - h_R = b\left(\frac{Q}{2\pi D}\right)^2\left[\frac{1}{r_w} - \frac{1}{R}\right] + \frac{aQ}{2\pi D}\log_e\left(\frac{R}{r_w}\right)$$

The first part of head loss is due to turbulence and the second part is due to viscous resistance.

The specific recharge rate is thus decreased due to turbulence.

In case of a pumping well, for turbulent flow condition, the relation between hydraulic gradient and velocity is

$$i = -av + bv^2 \tag{5}$$

$$-Q_P = 2\pi rDv_r \tag{6}$$

$$v_r = \frac{a}{2b} - \frac{a}{2b}\left(1 + \frac{4bi}{a^2}\right)^{1/2} \approx \frac{-i}{a} + \frac{i^2 b}{a^3} \tag{7}$$

Incorporating (7) in (6)

$$-Q_P = 2\pi r D\left(\frac{a}{2b} - \frac{\sqrt{a^2 + 4b\frac{dh}{dr}}}{2b}\right) \tag{8}$$

or

$$\left(a + \frac{bQ_p}{\pi r D}\right)^2 = a^2 + 4b\frac{dh}{dr} \tag{9}$$

or

$$\left(\frac{Q}{2\pi D}\right)^2 b\frac{dr}{r^2} + \frac{aQ}{2\pi D}\frac{dr}{r} = dh \tag{10}$$

Integrating

$$h = -\left(\frac{Q}{2\pi D}\right)^2 \frac{b}{r} + \frac{aQ}{2\pi D}\ln r + A \tag{11}$$

Applying the boundary condition

$$h_R - h_w = b\left(\frac{Q}{2\pi D}\right)^2\left(\frac{1}{r_w} - \frac{1}{R}\right) + \frac{aQ}{2\pi D}\ln\frac{R}{r_w} \tag{12}$$

The first part of the head loss is due to turbulence. The specific capacity of the pumping well is thus reduced due to turbulence. The equation shows that well loss can be reduced in case of large diameter well.

Ground Water Flow Equation

Groundwater flow aquifers is very often described by equations obtained by combining Darcy's law, as a dynamic equation with the continuity equation, adapted to the specific hydro geological conditions of the system being investigated.

Consider a control volume with dimension $\Delta x, \Delta y, \Delta z$ at point x, y, z (Fig.2(a)). Let the flow be three-dimensional. Let u, v, w are components of Darcy velocity which are functions of independent space variables x, y, z, and time parameter t. Unless otherwise specified the space derivative of components of Darcy velocity are bounded every where. Let the hydraulic head at point x, y, z and at time *t* be *h* and at time $t + \Delta t$ the hydraulic head be $h + \Delta h$.

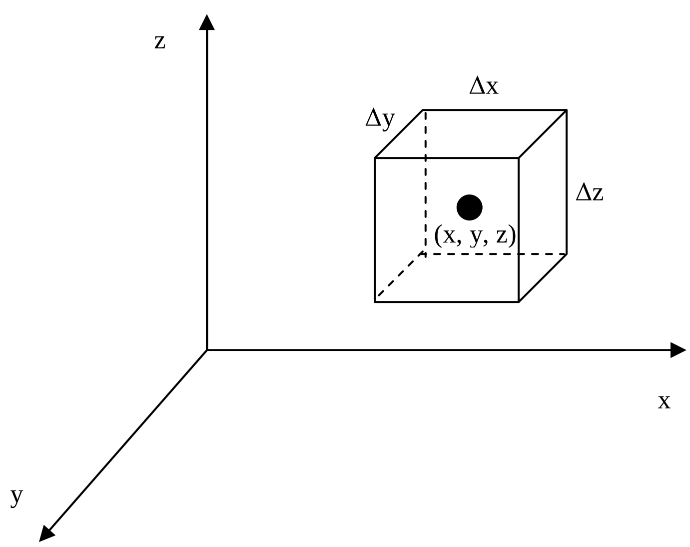

Fig. 2 (a) -A Control volume in a three-dimensional flow domain

Performing mass balance over a time period Δt within the control volume

Inflow - Outflow = Change in storage

or

$$\left[u(x,y,z)-\frac{\partial u}{\partial x}(\Delta x/2)\right]\Delta y\ \Delta z\ \Delta t+\left[v(x,y,z)-\frac{\partial v}{\partial y}(\Delta y/2)\right]\Delta x\ \Delta z\ \Delta t$$

$$+\left[w(x,y,z)-\frac{\partial w}{\partial z}(\Delta z/2)\right]\Delta x\ \Delta y\ \Delta t$$

$$-\left[u(x,y,z)+\frac{\partial u}{\partial x}(\Delta x/2)\right]\Delta y\ \Delta z\ \Delta t-\left[v(x,y,z)+\frac{\partial v}{\partial y}(\Delta y/2)\right]\Delta x\ \Delta z\ \Delta t$$

$$-\left[w(x,y,z)+\frac{\partial w}{\partial z}(\Delta z/2)\right]\Delta x\ \Delta y\ \Delta t$$

$$= S\Delta x\Delta y\Delta z\Delta h \qquad (1)$$

where S=specific storage i.e. volume of water that is released (or taken into storage) for unit volume of aquifer for unit drop (or rise) in piezometric surface. Simplifying equation (1) reduces to

$$-\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right]= S\frac{\partial h}{\partial t} \qquad (2)$$

Incorporating Darcy's law (i.e. $u=-k_x\frac{\partial h}{\partial x};\ v=-k_x\frac{\partial h}{\partial y};\ w=-k_z\frac{\partial h}{\partial z}$) in equation (2)

$$\left[k_x\frac{\partial^2 h}{\partial x^2}+k_y\frac{\partial^2 h}{\partial y^2}+k_z\frac{\partial^2 h}{\partial z^2}\right]= S\frac{\partial h}{\partial t} \qquad (3)$$

k_x, k_y, k_z = coefficients of permeability in principal permeability directions x, y, z respectively.

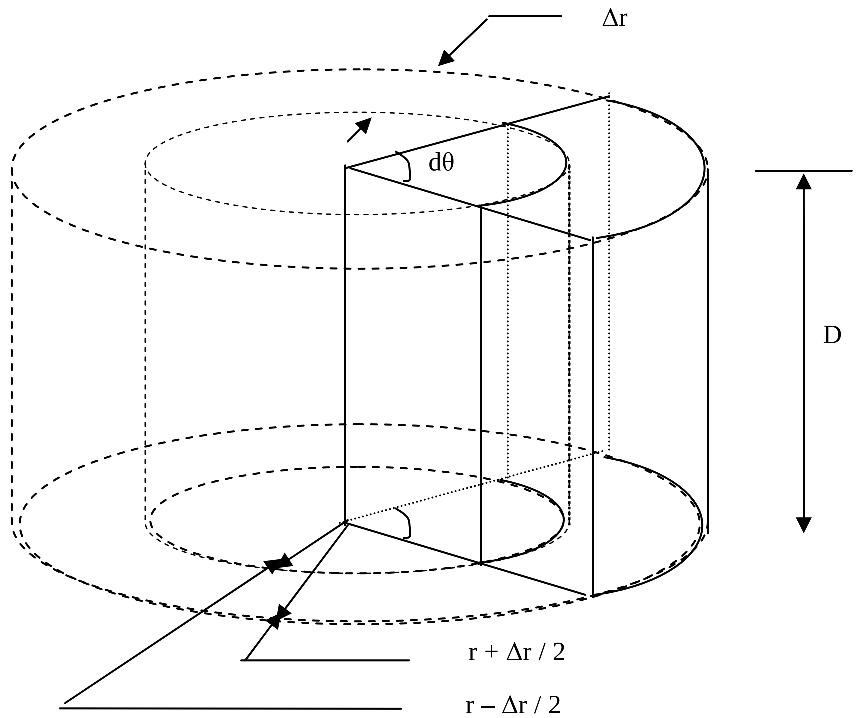

Fig. 2 (b). A control volume in axis symmetry radial flow domain

For two dimensional axis symmetry radial flow in a confined aquifer (Fig. 2 (b)) the governing equation for unsteady groundwater flow is derived as follows: Consider a control volume bounded by cylindrical surfaces at $r - \Delta r/2$, $r + \Delta r/2$, vertical surfaces at θ and at $\theta + \Delta\theta$ and horizontal impervious surfaces at the base and top of the confined aquifer. Let at point (r, θ), the radial velocity be v_r and hydraulic head be h.

Inflow= $(r - \Delta r/2)\Delta\theta D\left[v_r - \frac{\partial v_r}{\partial r}(\Delta r/2)\right]\Delta t$.

Outflow=. $(r + \Delta r/2)\Delta\theta D\left[v_r + \frac{\partial v_r}{\partial r}(\Delta r/2)\right]\Delta t$

Change in storage= $S\ r\ \Delta\theta\ \Delta r\ D\ \Delta h$.

For a homogeneous isotropic medium k_x=k_y=k_r .

Performing mass balance and incorporating Darcy's law $v_r = -k_r \frac{\partial h}{\partial r}$

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{\phi}{T}\frac{\partial h}{\partial t} \qquad (4)$$

where $T = k_r D$ = transmissivity; $\phi = SD$ =storage coefficient.

The position of water table or piezometric surface is generally measured from a high datum instead of measuring its position from the base of the aquifer. For an aquifer initially at rest, the initial piezometric surface is selected as a high datum. At any time, at a location, the sum of draw down in piezometric surface measured from high datum and height of the surface measured from low datum is constant i.e s+h = a constant. The above equation can be expressed as:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{\phi}{T}\frac{\partial s}{\partial t} \qquad (5)$$

When a fully penetrating well in an extensive homogeneous and isotropic confined aquifer is pumped, axis symmetry radial flow condition prevails and the unsteady flow is governed by above equation. Drawdown consequent to pumping water from a well of very small radius at a constant rate is predicted using Theis' solution for the above differential equation satisfying the initial condition $s(r,0)=0$; and the boundary conditions (i) $s(\infty,t)=0$; (ii) $2\pi\, rD\left(k\frac{\partial s}{\partial r}\right)_{r\to 0}=-Q_P$. The first boundary condition is a constant head boundary condition and is known as Dirichelt type and the second boundary condition is known as Cauchy type boundary condition. Theis non-equilbrium equation is:

$$s(r,t)=\frac{Q_P}{4\pi T}\int_{\frac{\phi R^2}{4Tt}}^{\infty}\frac{e^{-v}}{v}dv=\frac{Q_P}{4\pi T}W(U) \tag{6}$$

$W(U)$ is known as Theis Well Function.

$$W(U)=\int_U^{\infty}\frac{e^{-v}}{v}dv=-0.577216-\ln(U)-\sum_{n=1}^{\infty}\frac{(-1)^n U^n}{nn!}$$

$U=\frac{\phi\, r^2}{4T\ t}$; r = distance of the piezometer from the well, t=time since pumping starts. If pumping stops after time t_p, the residual drawdown at time t is given by:

$$s(r,t)=\frac{Q_P}{4\pi T}\left[W\left(\frac{\phi\, r^2}{4Tt}\right)-W\left(\frac{\phi\, r^2}{4T(t-t_p)}\right)\right] \tag{7}$$

If unit volume of water is pumped in a time period of Δt, $t=n\Delta t$, n = an integer, the drawdown $\delta(r,n\Delta t)$ is:

$$\delta(r, n\Delta t) = \frac{1}{4\pi T\Delta t}\left[W\left(\frac{\phi\, r^2}{4Tn\Delta t}\right) - W\left(\frac{\phi\, r^2}{4T(n\Delta t - \Delta t)}\right)\right] \tag{8}$$

If pumping rate is not uniform or pumping is not continuous, the drawdown is computed using the relation

$$s(r, n\Delta t) = \sum_{\gamma=1}^{n} Q_V(\gamma)\delta\{r, (n - \lambda + 1)\Delta t\} \tag{9}$$

where $Q_V(\gamma)$=volume of water withdrawn during γ^{th} Δt time unit.

In case of a well in a confined aquifer of infinite extent under continuous pumping steady flow condition as indicated by Theis non equilibrium formula is never reached. If pumping at a constant rate is continued for a long time and the well is located near a large surface water body, a steady state flow condition is reached. In that case, $\frac{\partial h}{\partial t} = 0$. The governing equation of flow is

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \tag{10}$$

For radial-axis-symmetry flow condition (i.e. when a fully penetrating well is located at center of an island) the flow equation reduces to

$$\frac{d^2 h}{dr^2} + \frac{1}{r}\frac{dh}{dr} = 0 \tag{11}$$

The boundary conditions to be satisfied are: $h(r_w) = h_w; h(R) = h_R$ where r_w=radius of well and *R*=radius of the circular island.

Equation (11) can be rewritten as:

$$\frac{d}{dr}\left(r\frac{dh}{dr}\right) = 0 \qquad (12)$$

Integrating

$$r\frac{dh}{dr} = C \qquad (13)$$

where C is a constant to be determined.

Again integrating

$$h = C \ln r + A \qquad (14)$$

At $r = r_w, h = h_w$ (i.e. at the well face the hydraulic head $= h_w$). Applying this condition

$$A = h_w - \ln r_w \qquad (15)$$

and

$$h - h_w = C \ln \frac{r}{r_w} \qquad (16)$$

At any radial distance the radial flow is constant as the flow is under steady state condition.

Hence,

$$2\pi r D\left(-k\frac{dh}{dr}\right) = -Q_P \qquad (17)$$

$$C = \left(r \frac{dh}{dr} \right) = \frac{Q_P}{2\pi T} \tag{18}$$

Incorporating constant C in equation (16)

$$h - h_w = \frac{Q_P}{2\pi T} \ln \frac{r}{r_w} \tag{19}$$

or

$$Q_P = 2\pi T \left[h - h_w \right] / \ln \left(\frac{r}{r_w} \right) \tag{20}$$

This equation relating steady flow with hydraulic head is known as Theim equation.

For unconfined aquifer, applying Dupuit assumptions (i.e. (a). for small inclination of the top flow lines the flow lines are horizontal and hence, the equipotential lines are vertical; (b) the gradient at any section is given by the slope of the top flow lines and the gradient is invariant with depth), the flow at any section is:

$$-Q_P = 2\pi\, rh \left(-k \frac{dh}{dr} \right) \tag{21}$$

Separating the variable, integrating and applying the head boundary condition at the well

$$Q_P = 2\pi\, k \left[h^2 - h_w{}^2 \right] / \ln \left(\frac{r}{r_w} \right) \tag{22}$$

Aquifer Tests

The hydraulic properties of an aquifer (transmissivity and storage coefficient) can be determined by pumping a well at constant discharge and observing the drawdown at an observation well for a period of time. For a confined aquifer the Theim steady state equation yields only the transmissivity

$$T = (Q_P \ln\frac{r_2}{r_1})/[2\pi(s_1 - s_2)] \tag{23}$$

s_1 and s_2 are observed drawdown at distance r_1 and r_2 from the well.

The transmissivity and storage coefficient are estimated using Theis' non equilbrium formula i.e. using Theis' type curve (fig.3(a),(b)). From equation (6)

$$s(r,t) = \frac{Q_P}{4\pi T}W(U) \tag{24}$$

Taking logarithm of terms on either side

$$\log_{10}[s(r,t)] = \log_{10}[\frac{Q_P}{4\pi T}] + \log_{10}[W(U)] \tag{25}$$

From $U = \dfrac{\phi\, r^2}{4T\, t}$

$$\frac{t}{r^2} = \frac{\phi}{4TU} \tag{26}$$

Taking logarithm of terms on either side

$$\log_{10}\left(\frac{t}{r^2}\right) = \log_{10}\left(\frac{\phi}{4T}\right) + \log_{10}\left(\frac{1}{U}\right) \qquad (27)$$

Considering equations (25) and (27) it is stated that a graph of W(U) versus (1/U) would match with the graph of s versus t when the latter is moved on the former (type curve) keeping the abscissas or the ordinates parallel to each other. The time draw down curve and the type curve should be plotted on same type of double log paper. After matching, a convenient point is selected on the type curve; $U^*, W(U^*)$ and corresponding $s^*, \frac{t^*}{r^2}$ are noted.

$$T = \frac{Q_P}{4\pi\ s^*} W\left(U^*\right) \qquad (28)$$

$$\phi = \frac{4Tt^* U^*}{r^2} \qquad (29)$$

One may find aquifer parameters using a graph of s(r,t) versus $\frac{r^2}{t}$ and matching with type curve of W(U) versus (U) . However the former is more convenient than the latter.

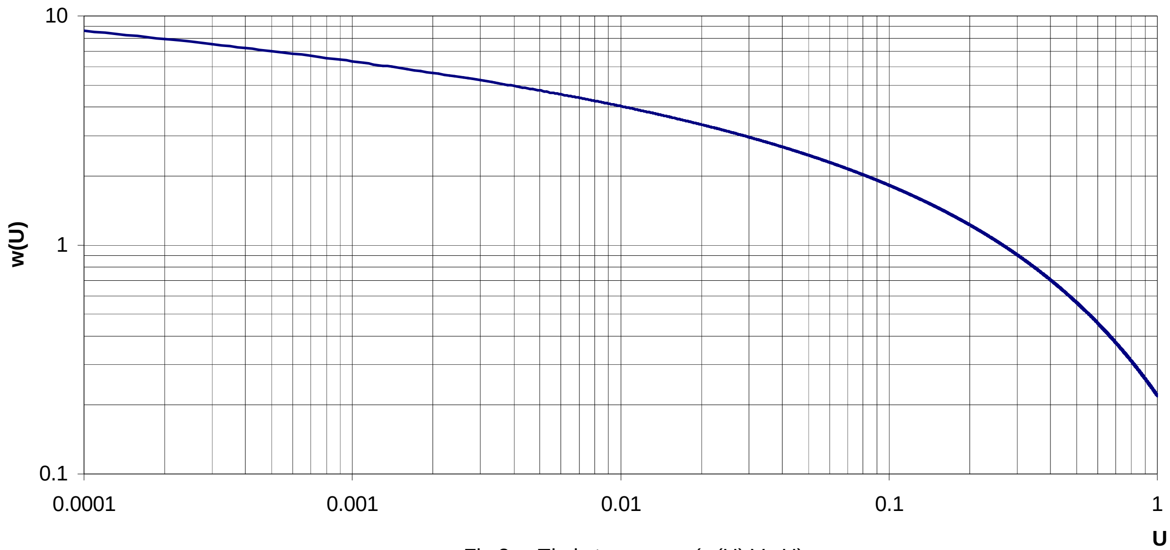

Fig 3 a. Theis type curve (w(U) Vs U)

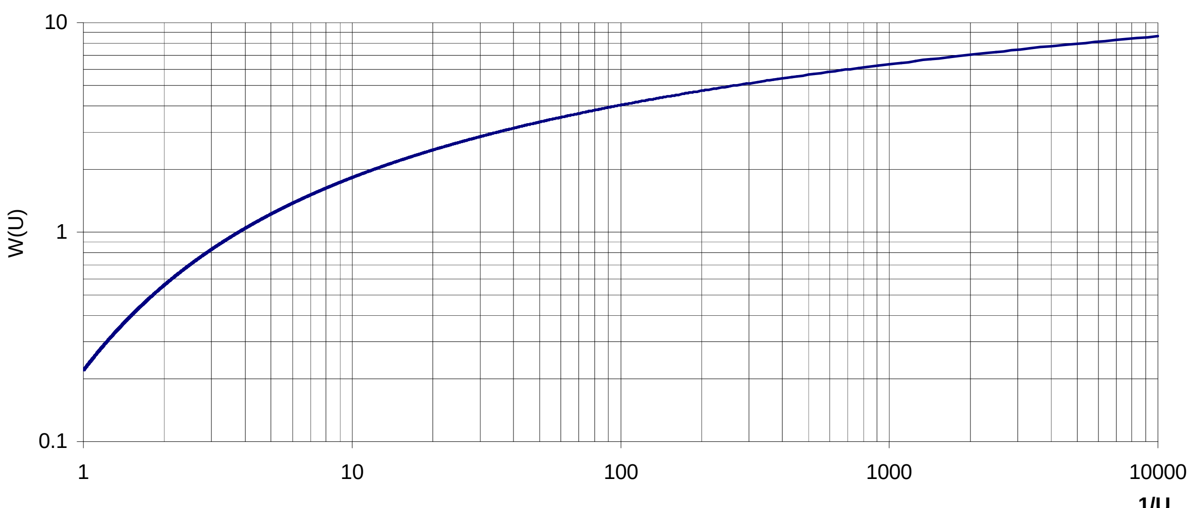

Fig 3 b. Thesis type curve (w (U) vs. 1 / U)

Stream Aquifer Inter action

A stream often forms the boundary of an aquifer. If the stream is a partially penetrating one, exchange of flow between the stream and the aquifer is to be defined by relating stream stage and head in the aquifer adjacent to the stream. The linear relation between exchange flow rate and head difference is given by:

$$Q = \Gamma_r \quad (h_r - h_a)$$

in which Q is the flow from the stream to the aquifer and Γ_r is a constant known as reach transmissivity (Morel-Seytoux et al,1979), h_r=head in the stream forming the boundary, h_a=head in the aquifer very near to the stream (h is unknown unless observed). The reach transmissivity constant depends on the hydraulic conductivity of the aquifer medium, stream geometry, location of the observation point at which head h_a is measured. The reach transmissivity constant is derived for a steady state flow condition and assuming an unsteady state is a succession of steady states, the constant is used for solving an unsteady flow problem. The reach transmissivity constant for a partially penetrating stream of large width (Fig. 4 (a)) is presented here.

The flow being in steady state, the flow rate at any section in the aquifer beyond the stream bank is same and equal to *q*. The flow can be quantified only for an observed value of *h(l)* at any distance *l* from the stream bank. The variation of $q/[k(h_r - h(l))]$ with l/D_2 for various D_1/D_2 is presented in Fig.4 (b). The ratio $q/[k(h_r - h(l))]$ is a dimensionless reach transmissivity constant (Morel-Seytoux et al 1978) for a stream reach of unit length. As seen from figure the reach transmissivity constant is a function of l and D_1/D_2.

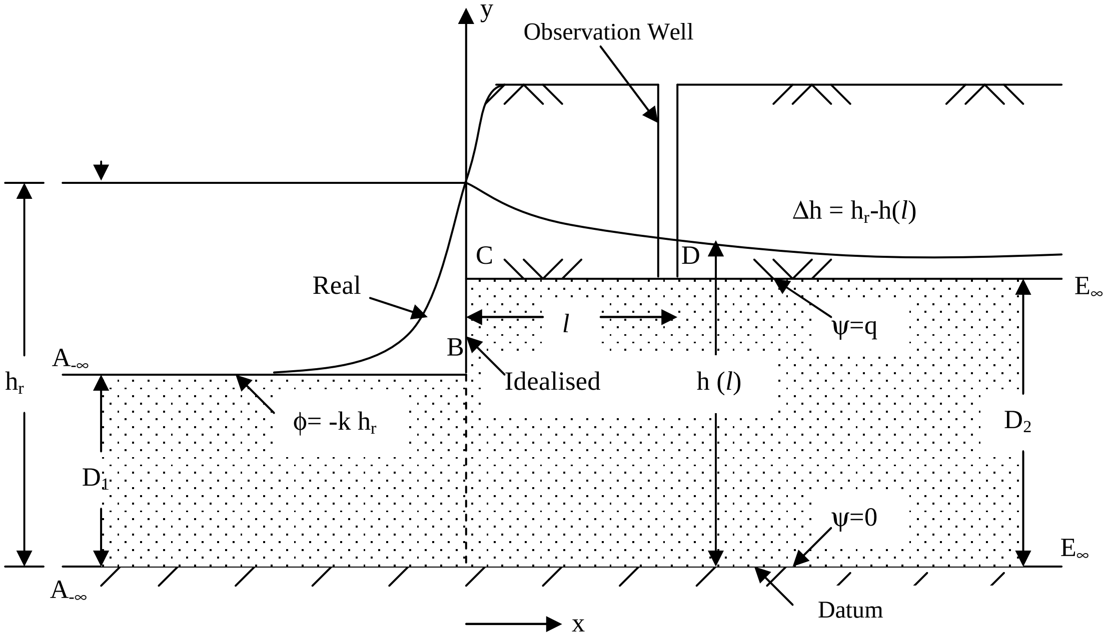

Fig. 4 (a) Physical flow domain in z (=x+iy) plane

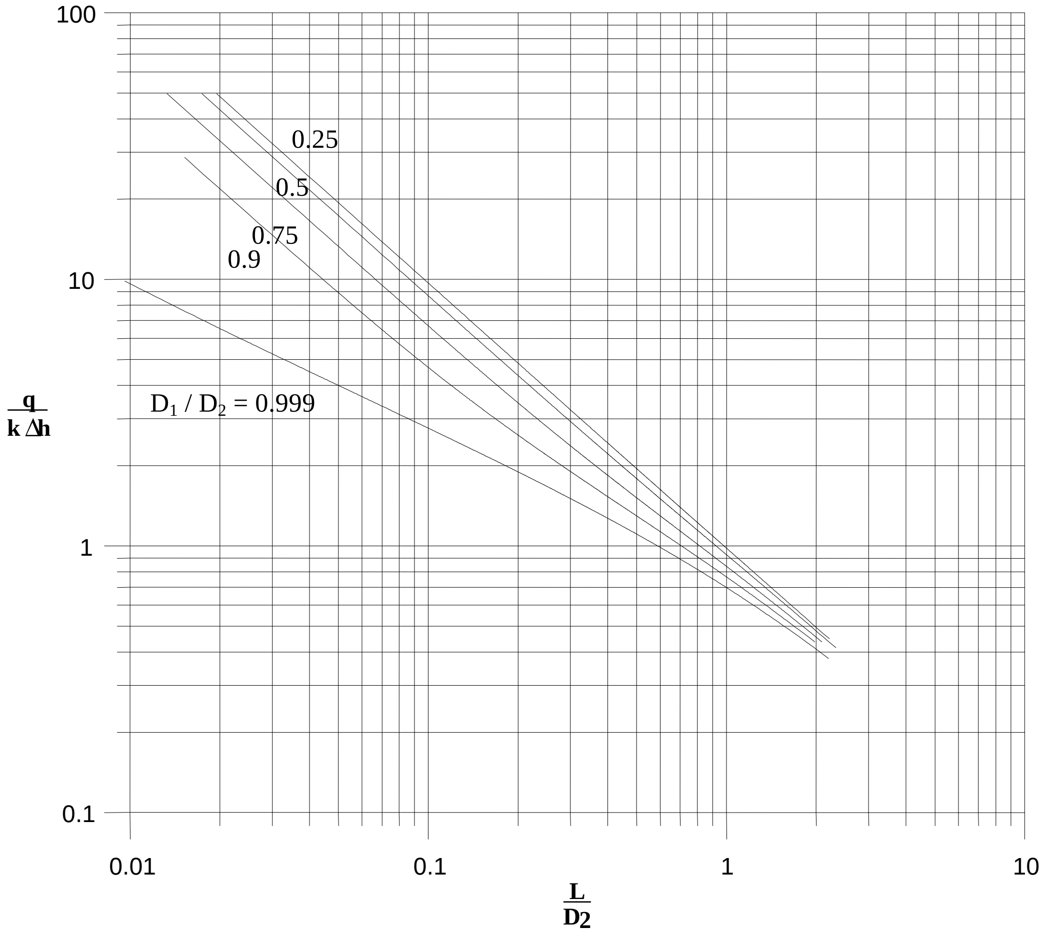

Fig. 4 (b). Variation of q / (k Δh) with distance from stream bank for a partially penetrating stream

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