MODELING OF GROUNDWATER FLOW SYSTEM

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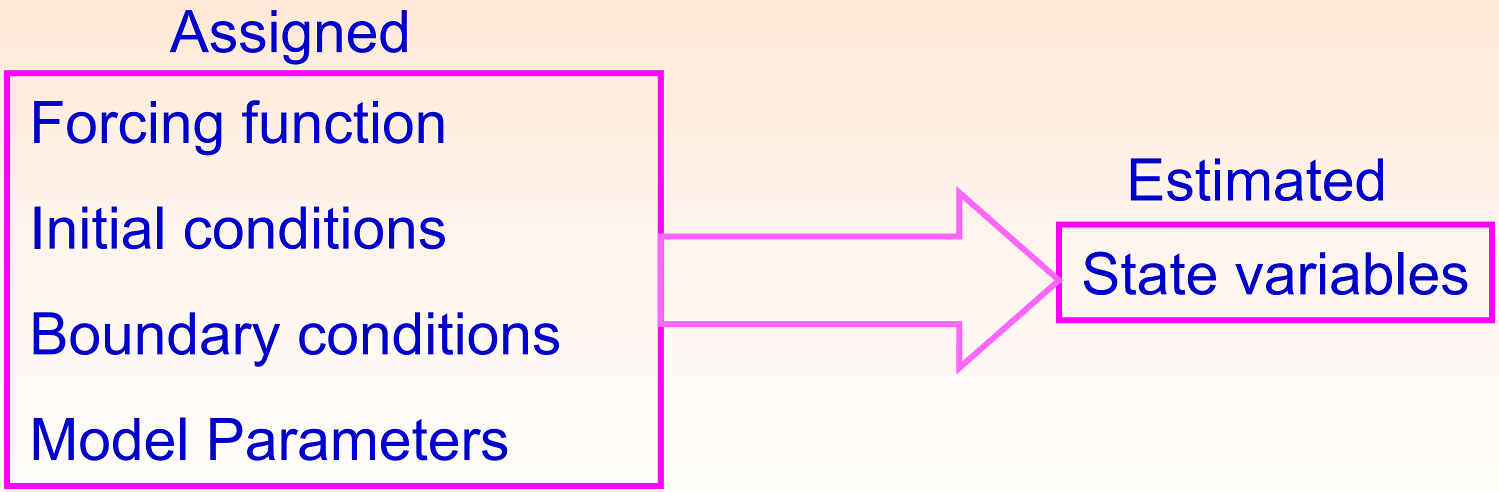
Groundwater Flow Modeling

- ❖ Numerical experiments on a groundwater flow model

Groundwater Flow Model



State Variables

- ❖ Describe the “state” of a system
- ❖ Divided in two categories
  1. Mandatory state variable
  2. Problem specific/ derived state variables

Mandatory State Variable

(Direct End-product From Modeling)

Space and time distribution of head

Problem- Specific State Variables
(Derived from Mandatory State Variable)

- Depth to water table
- Stream- aquifer interflows
- Sea water intrusion
- Static reserve etc.

Forcing Function

- Withdrawals
- Recharge
- Evapotranspiration from the saturated zone

Initial conditions

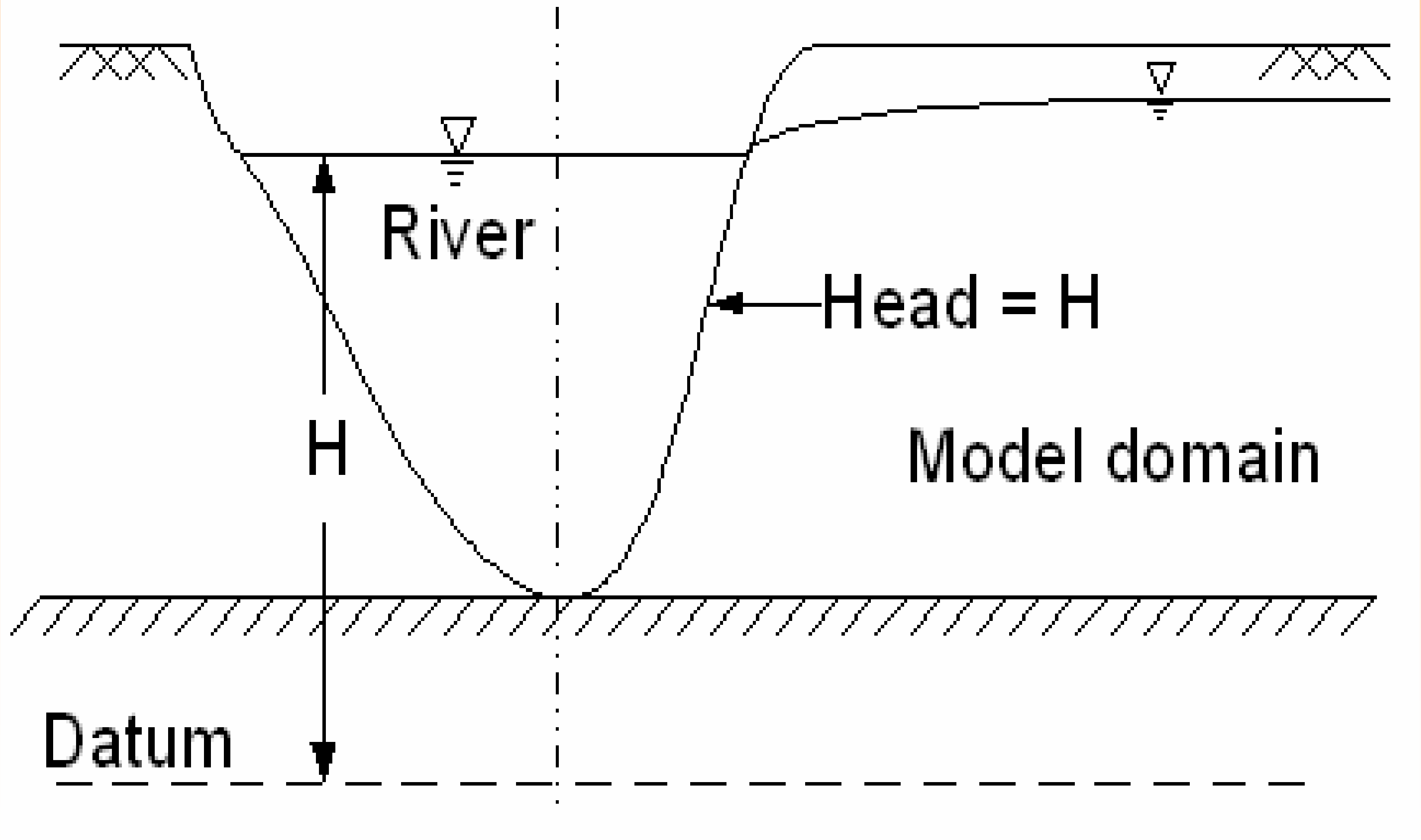
- Spatial distribution of the head at the instance when the assigned forcing function commences to act

Boundary Conditions

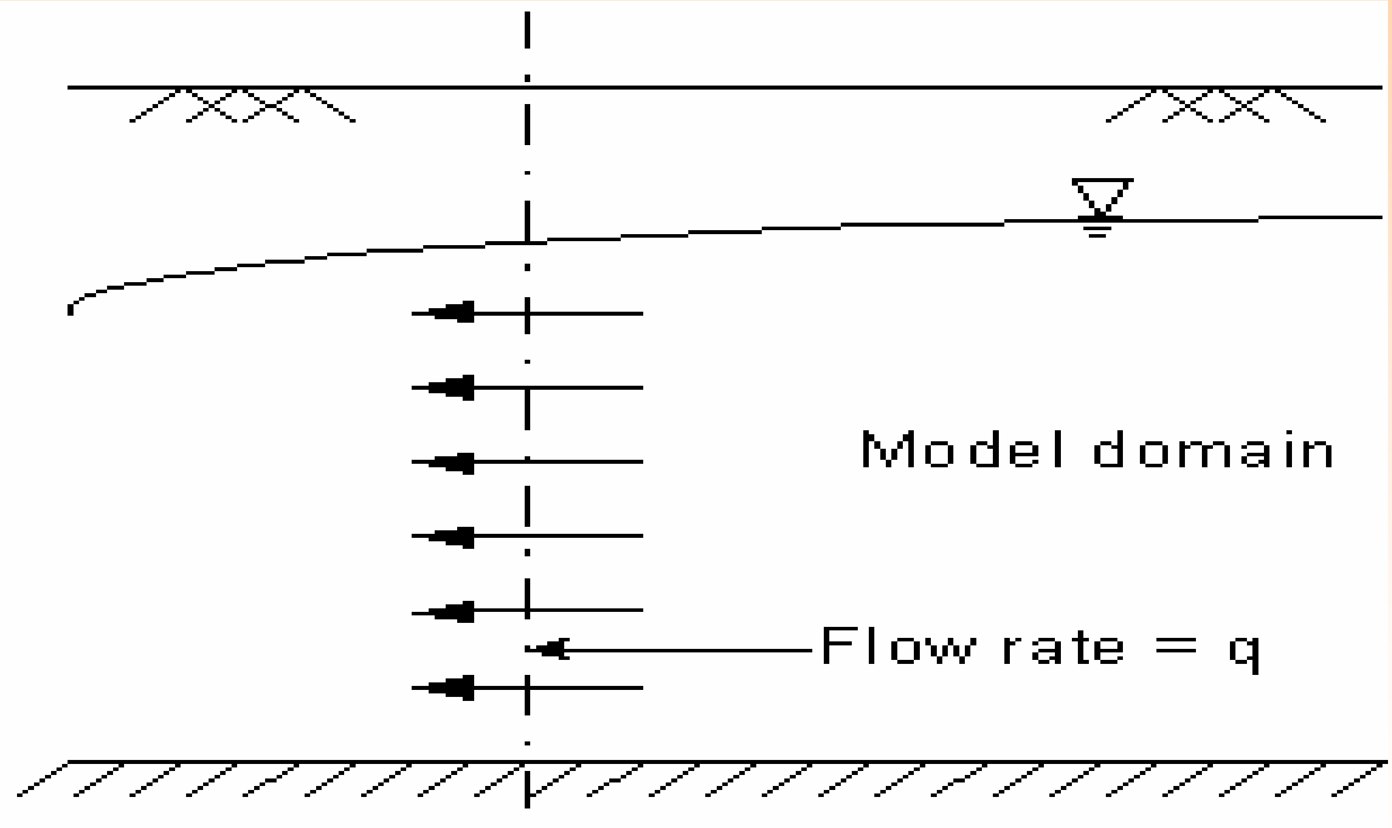
- Head assigned
- Flow assigned

Few Examples of Boundary Conditions

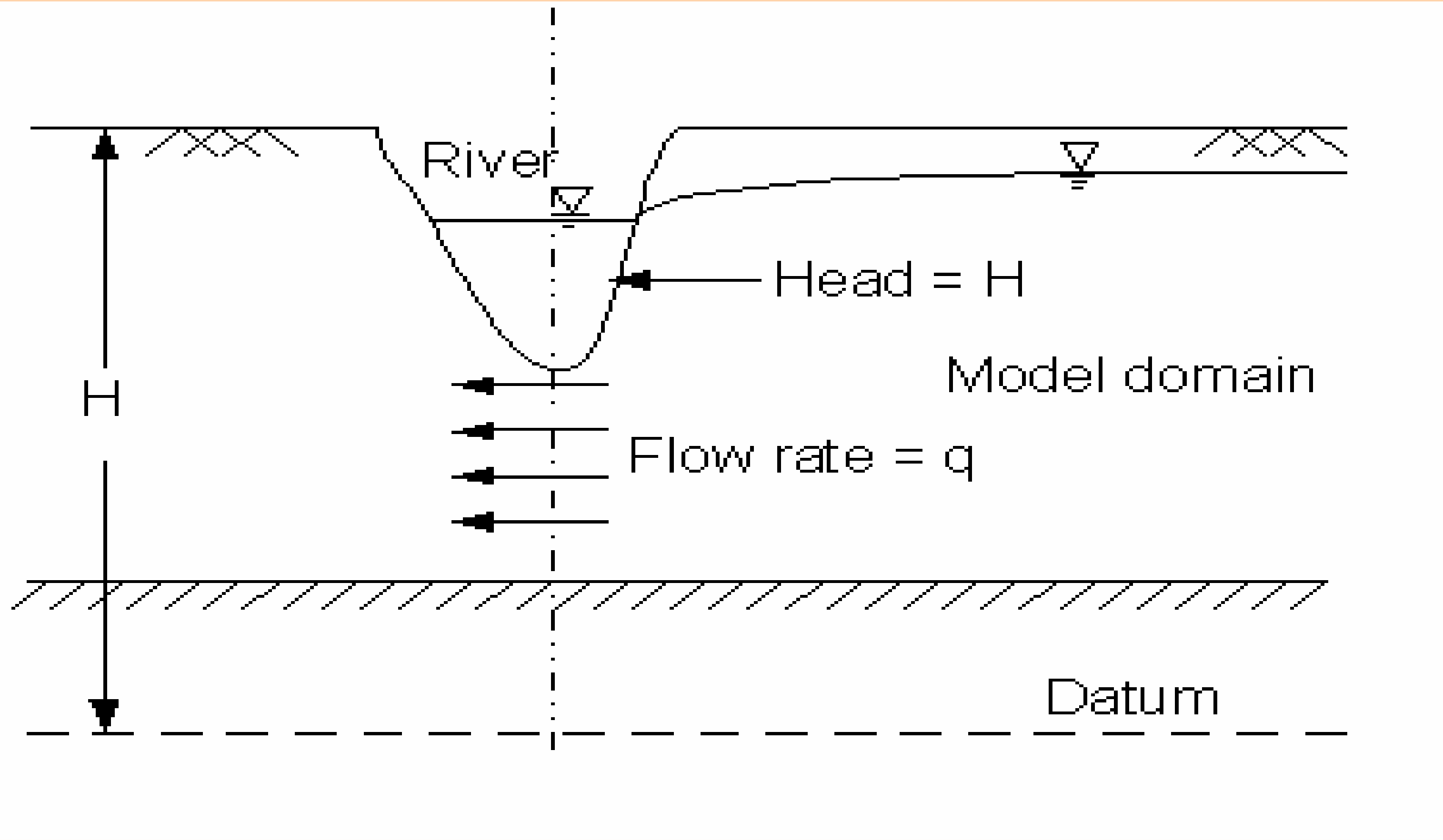
Head Assigned Boundary Condition



Flow Assigned Boundary Condition



Head/ Flow Assigned Boundary Conditions



Model Parameters

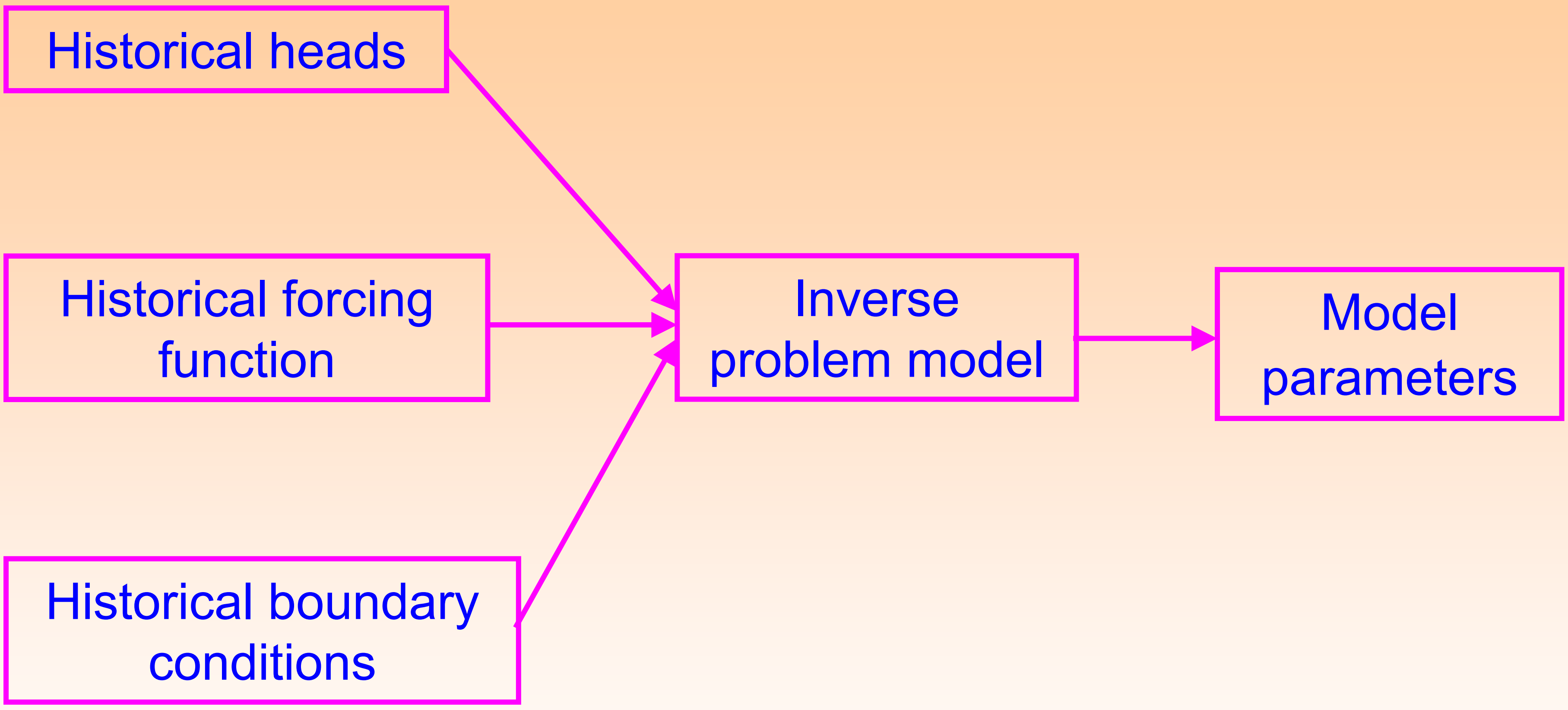
Represent Aquifer- Characteristics in the Model

Types of the Model Parameters

- Flow Parameters
- Storage Parameters

Estimation of Model Parameters

- ❖ Pumping Tests
- ❖ Inverse Modeling



Inverse problem

Components of a GW Flow Model

- Governing equation
- An algorithm to solve the equation for the mandatory State Variable
- Algorithms to compute the problem- specific state variables
- Computer codes to implement the algorithms

Governing Equation

- Mass Balance (Continuity) Equation
- Auxiliary Relations

Continuity Equation

Change of the storage = Inflow - Outflow

- Types of inflows/ outflows
  1. Gradient driven flow
  2. Forcing function driven flow

Domain for Continuity Equation

Infinitesimally Small Length/ Area/ Volume

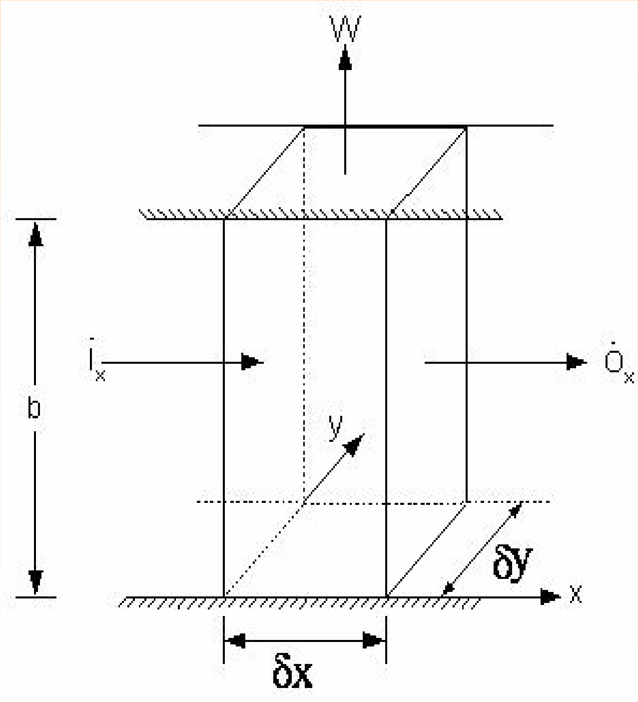
An Example

Horizontal flow in a confined aquifer

Domain: Infinitesimally small area in horizontal plane, extending over the aquifer thickness

Writing continuity eqn. over the selected domain….

Gradient driven flow rates in x- direction:



$$\dot{I}_x = -K_{xx} \frac{\partial h}{\partial x} b \delta y \quad = -T_{xx} \frac{\partial h}{\partial x} \delta y$$

$$\dot{O}_x = -\left[T_{xx} \frac{\partial h}{\partial x} + \frac{\partial}{\partial x}\left(T_{xx} \frac{\partial h}{\partial x} \right) \delta x \right] \delta y$$

$$\dot{I}_x - \dot{O}_x = \frac{\partial}{\partial x}\left(T_{xx} \frac{\partial h}{\partial x} \right) \delta x \, \delta y$$

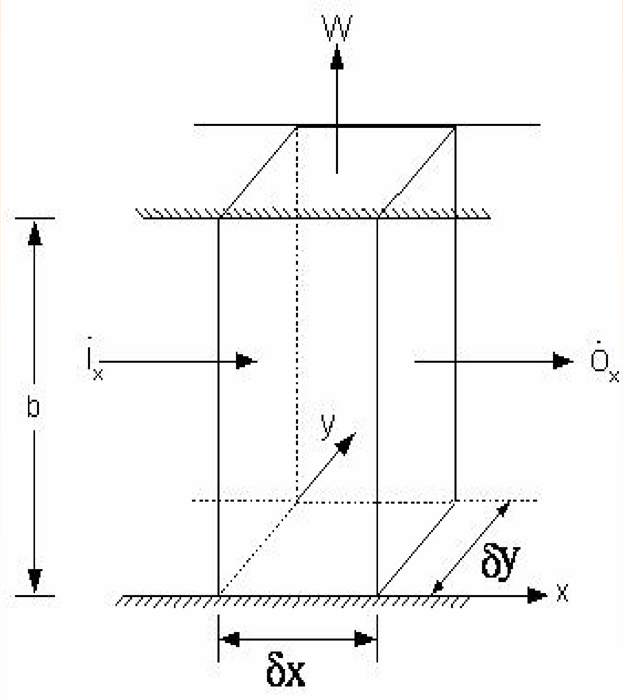
Similarly $\dot{I}_y - \dot{O}_y = \frac{\partial}{\partial y}\left(T_{yy} \frac{\partial h}{\partial y} \right) \delta x \, \delta y$

Total gradient driven (Inflow - Outflow) rate

$$= \frac{\partial}{\partial x}\left(T_{xx} \frac{\partial h}{\partial x} \right) \delta x \, \delta y + \frac{\partial}{\partial y}\left(T_{yy} \frac{\partial h}{\partial y} \right) \delta x \, \delta y$$

Forcing function driven out flow rate = W δx δy

$$\text{Total}\ (\dot{I}-\dot{O}) = \frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)\delta x\delta y + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)\delta x\delta y - W\,\delta x\delta y$$



$$\text{Rate of change of storage} = \frac{\partial h}{\partial t}S\delta x\delta y$$

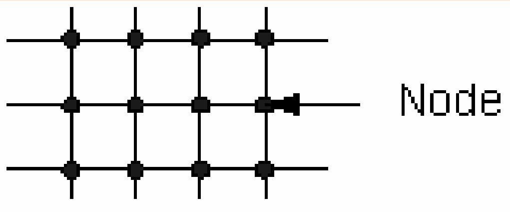
Resulting governing differential equation:

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) - W = S\frac{\partial h}{\partial t}$$

Solution Algorithms

Finite Difference Method

❖ **Discretization of space**

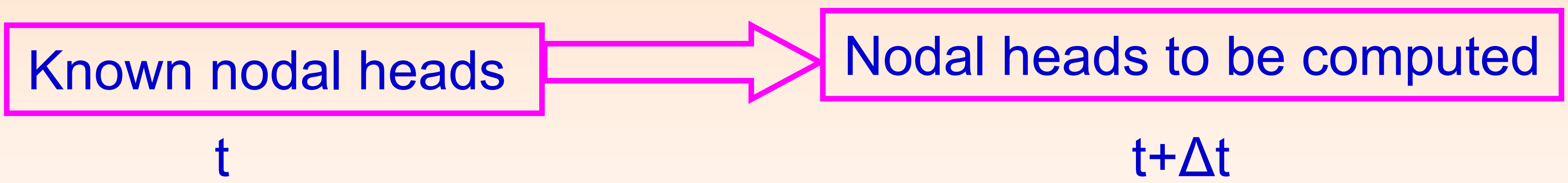


❖ **Discretization of time**

Simulation period is discretized in to finite number of discrete times

Marching in time domain

- Solution commences from the assigned Initial condition



FDM Strategy

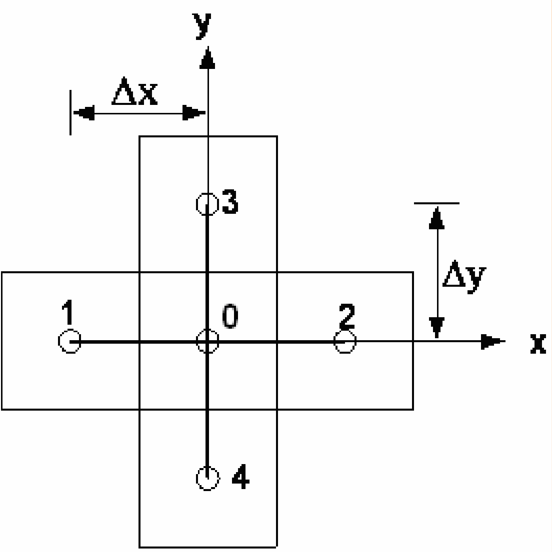
The governing differential equation is transformed into a determinate system of linear equations

An Example

Solution of the differential equation governing two dimensional horizontal flow in a confined aquifer

❖Interior nodes

The node "0" surrounded by four nodes "1", "2", "3" and "4" as shown below.



The space and time derivatives of h at node 0 :

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)=\frac{1}{\Delta x}\left[\frac{h_2-h_0}{\Delta x}\left(\frac{T_0+T_2}{2}\right)-\frac{h_0-h_1}{\Delta x}\left(\frac{T_0+T_1}{2}\right)\right]$$

$$\frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right)=\frac{1}{\Delta y}\left[\frac{h_3-h_0}{\Delta y}\left(\frac{T_0+T_3}{2}\right)-\frac{h_0-h_4}{\Delta y}\left(\frac{T_0+T_4}{2}\right)\right]$$

$$S\frac{\partial h}{\partial t}=S_o\frac{h_0-h_0^i}{\Delta t}$$

This leads to following linear equation in terms of five unknown heads

$$A(h_o)+B(h_1)+C(h_2)+D(h_3)+E(h_4)=F$$

❖Boundary nodes

An additional linear equation is obtained for each boundary node by invoking the respective known boundary conditions

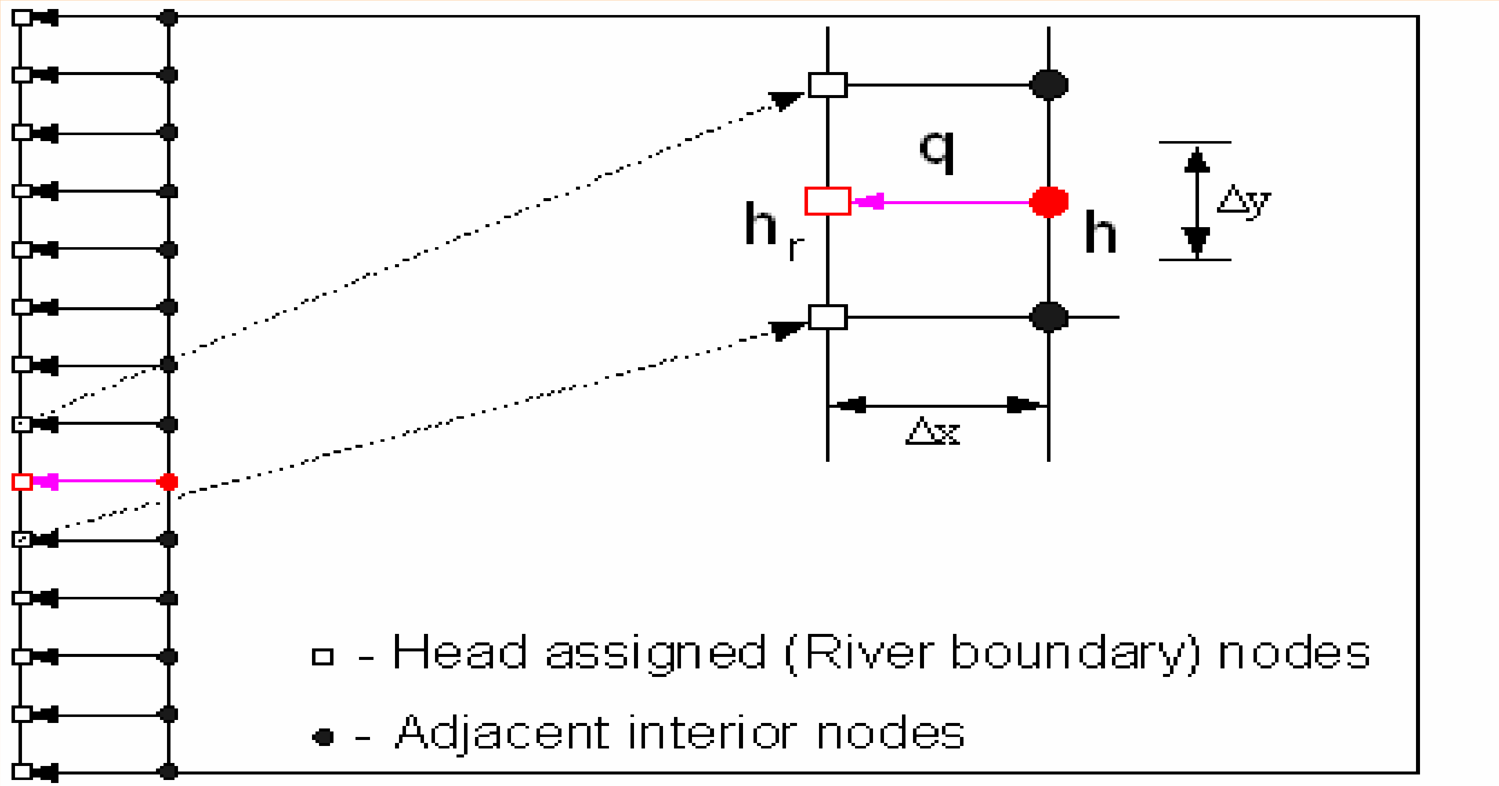
Solution of Linear Equations

Dedicated algorithms

Computation of Problem- Specific State Variables

From the computed head distributions

Computation of Problem- Specific State Variables – An Example



Derivation of stream-Aquifers inter flows

$$\text{Discharge (q)} = \frac{h - h_r}{\Delta x} T \, \Delta y$$

Feasibility Checks

1. Identify the aquifer system
2. Quantify the proposed pumping/ recharge pattern
3. Identify the constraints and the corresponding state variables of the groundwater system
4. Formulate the nodal forcing functions by adding algebraically the proposed pumping/ recharge and other “natural” source/ sink terms
5. Project the nodal heads and hence the relevant state variables
6. Check feasibility

Uncertainty in Projections

❖ Assumptions like horizontality of the flow, uniqueness of the parameters, boundary conditions, spatial distribution of the aquifer parameters etc. introduce an uncertainty in the model projections.

Conclusion

GWF modeling is essentially a tool to project the state variables of the groundwater system